THE LIGHTNING EMPIRICIST

Advocating electronic models, at least until livelier instrumentalities emerge

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IMPEDANCE & ADMITTANCE TRANSFORMATIONS
using Operational Amplifiers

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I. Transadmittance, Transimpedance, Positive and Negative Self-Impedance through Active Circuits, including references to:

Photomultiplier and ion-current amplifiers;
Current sources and generators; and
Negative resistors and capacitors for dynamic compensation with Single-Ended, Differential, and Inverted Amplifiers

This is to be an abbreviated contemplation of a subject, the rigorous or exhaustive treatment of which might occupy a veritable volume. Our modest ambition here is to present a few unusual principles, with possible circuit applications, which might stimulate the thoughtful Reader to increase his rate of consumption of operational amplifiers.

INVERTING AMPLIFIERS

The conventional ideal inverting circuit is often simply described as one which maintains a voltage null and a current balance at its summing point, or negative input terminal, by feedback around a dc amplifier having high *power* gain. Such a circuit is exhibited in Figure 1. It is immediately seen that if the voltage null and current balance are well maintained, the output voltage is determined solely by the total input current and the feedback impedance; and the input current is the sum of the independent input currents, each determined solely by its input voltage and impedance. The first order effects of discrepancy between theory and practice are described elsewhere.

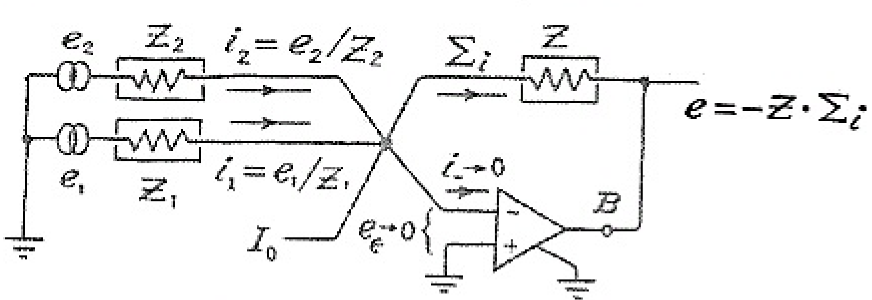

Fig. 1 Ideal Single-ended Operational Amplifier

These relationships immediately suggest, to one attuned and vigilant, that current from a current source, or charge from a charge source, can be converted into voltage at a specified level by proper choice of feedback impedance, and that an input voltage can be used to establish the current through an element connected in the feedback path of the amplifier. At first blush one might think of the circuits of Figure 2, briefly described as follows:

In Figure 2a, a photomultiplier, an ion-producing circuit, or some other high-impedance source of current feeds its current directly into (or more often, sucks the current i directly from) the summing point of the amplifier. This is not cruel when one remembers that current sources prefer to deliver their current to voltage nulls, which appear as low impedances and which do not load. In the example shown, an additional large resistor R_0 establishes the magnitude of an opposing current. I_0, which may furnish an index level or threshold, or compensate for "dark current" or background. The sum of the currents flows through the feedback element and produces an output voltage e.

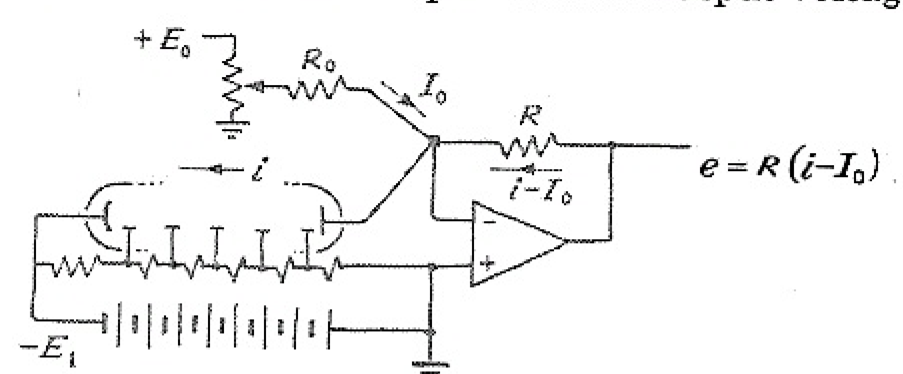

Fig. 2a Current-to-Voltage Transimpedance

The feedback element can be a linear resistor of virtually any magnitude for direct measurement of current; it can be a nonlinear resistor to produce a nonlinear function of current, thus compensating for nonlinearity in the transducer; or it can be a capacitor, to measure the integral of the input current, interpreted directly as charge, or of course the *average* input current over a given period. Because current is converted to voltage, usually substantial, the major error is current leakage, which determines the choice of amplifier. [Sales Manager's Note: P2 is considered an excellent choice.] Excitation voltage, E_1 is quite high, sometimes kilovolts; output is generally in volts; and erroneous millivolts or microvolts at the input e_ϵ produce millivolts or microvolts at the output, because voltage amplification is small. Common mode effects are

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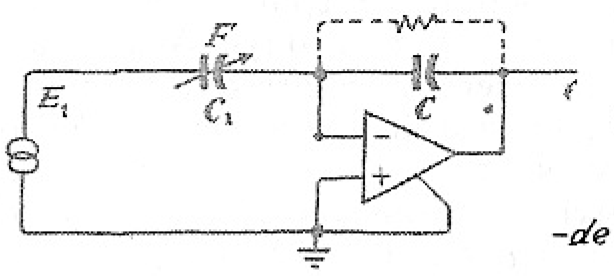
negligible, because the summing point is maintained close to zero. Voltage noise in the external circuit can be kept out without difficulty, because it is feasible to use grounded guards on the summing point without appreciably affecting response time. This approach to measurement contrasts favorably with the "potentiometric" approach, wherein (small) voltage is developed across a resistor, whose size is limited to avoid loading. The voltage, in being amplified, must cope with amplifier voltage offsets, common mode swing, and common mode circuit impedances, as well as voltage noise in the external circuit. [Sales Manager's Note: If you insist on the potentiometric approach, P2 is still an excellent choice, because of its low noise power, lower common mode rejection, less than 5×10^{-11} ampere offset current, and an even lower — 10^{-12} ampere — variation in that current.]



$$Q = C_1 E = -Ce$$
$$dQ = -C\,de$$
$$-de = E_1 \frac{dC_1}{C} + \frac{C_1}{C} dE_1$$

Fig. 2b Measuring Capacitive Sources
(Charge-to-Voltage Transimpedance)

In Figure 2b, an input capacitor is subjected to a varying stress. The operational amplifier preserves charge variation in the feedback element, and the output voltage measures the charge or the change in charge. In the case of the capacitive transducer (e.g. microphone), a fixed voltage stress E_1 is applied, and ΔQ is proportional to the change ΔC_1 in capacitance resulting from motion. Note the important properties of the operational amplifier that are especially useful in this application: the summing point is held at ground potential, hence voltage across the capacitor is truly constant; ac coupling is inherent, hence the dc excitation voltage is kept off the summing point; and the capacitance of shielded leads between the transducer and the amplifier does not significantly load the transducer, because that capacitance appears between summing point and ground.

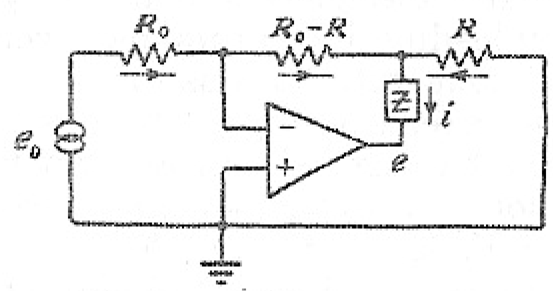
Question for the Reader to consider: Can one treat a piezoelectric transducer in this fashion? One might reason that the fixed stress (or "bias"), instead of being electrical, is mechanical, energy being stored in a spring instead of a battery, but that the basic response to stress is still a capacitance change, and hence change of charge, or a temporary flow of current.

In the case of the capacitor comparison bridge, there is no mechanical stress applied to the unknown capacitor C_1. The stress is electrical, in the form of an ac voltage e_1. An ac voltage is produced at the amplifier output, proportional to the ratio of input to feedback capacitance. A pair of equal external resistors can be used to provide a null measurement.

The principal drawback of the operational amplifier approach is that the dc leakage current will charge the feedback capacitor and eventually cause the amplifier to reach saturation. However, this possibility can be forestalled by using a large parallel resistor as a "leak" to establish the maximum voltage to which the output can depart. Note also that potentiometric approaches have the same "open grid" problem.

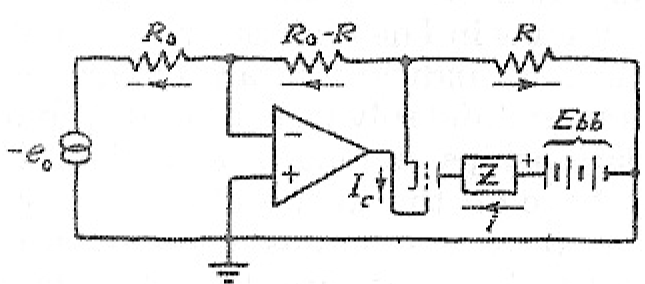
[Sales Manager's Note: Models P75, PP35A (especially), and P2 (for low frequency applications where capacitors are large and leakage important) are suitable for forward-looking devotees of solid state, noting the P2 and PP35A will serve potentiometrically as well. For those who still believe in thermionics, Models K2-W and SK2-V will do the job economically and well.]

In Figure 1, we saw a current generator perform the simplest job of its kind. The output voltage is adjusted by the amplifier to maintain whatever voltage is needed across the load element Z to keep the summing point at zero, or at a *virtual ground*. In this circuit, the voltage source is grounded, but the load cannot be grounded in the literal sense. The amplifier is single-ended: neither input nor output is used differentially. There is no net amplification of current, and the sources and the amplifier must both supply the current in Z. Any number of grounded sources may be used, and this current will be equal to the sum of the input currents. A current and voltage booster may be used at the point in the circuit marked "B" to provide any needed degree of augmentation. [Sales Manager's Note: Philbrick manufactures a number of popular boosters, and can advise on boosting circuitry in any case.]



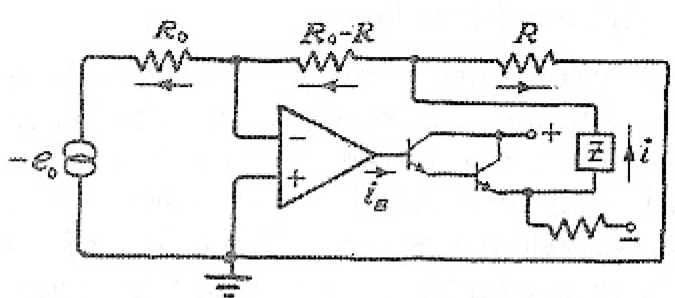
$$i = \frac{e_0}{R}$$
$$e_{MAX} = -i_{MAX} Z_{MAX} \left[1 + \frac{R(R_0 - R)}{R_0 Z_{MAX}}\right]$$

Fig. 3a Voltage to Current Transconductance



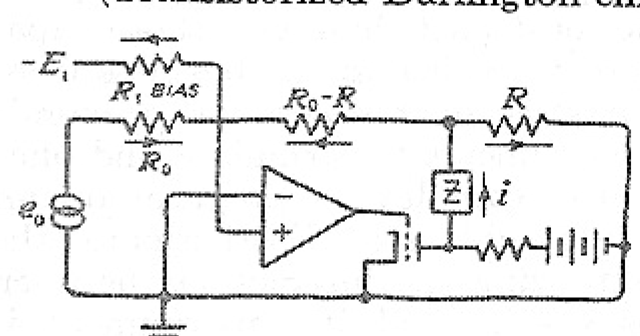
$$i = \frac{e_0}{R}$$
$$(I_c <<< i)$$

Fig. 3b Boosted Voltage to Current Transconductance
(Plate-loaded)



$$i = \frac{e_0}{R}$$
$$(i_B <<< i)$$

Fig. 3c Boosted Voltage to Current Transconductance
(Transistorized Darlington emitter follower)



$$i = \frac{E_1}{R} \frac{R_0}{R_1} - \frac{e_0}{R}$$
$$\frac{\partial i}{\partial e_0} = -\frac{1}{R}$$

Fig. 3d Boosted Voltage to Current Transconductance
(Shunt-controlled current)

In Figure 3, we see a number of possibilities for single-amplifier current generators using grounded sources. These circuits all have current amplification, both terminals of the current-receiving element (or
Continued on page 7

Amplifiers: continued from page 2

current load) are away from ground, and a few of the most popular do-it-yourself boosters are shown.

Figure 3a shows a circuit having current gain, in which the source needs supply only R_0/R_1 of the load current. Three typical booster arrangements are shown in Figures 3b and 3c, and an unusual *shunt* booster circuit is shown in Figure 3d. The shunt booster circuit is useful where high voltages would lead to large amounts of dissipation and undue voltage stress on circuit elements; but note that the amplifier polarity must be reversed, and that some care may be required to attain dynamic stability.

DIFFERENTIAL-INPUT AMPLIFIERS
(and single-ended equivalents using amplifier pairs)

It is possible to construct a quite simple current source, using a differential amplifier, a floating reference source which need supply no current, and an adjustable resistor. This current source will, within the ratings of the amplifier, supply current to a load whose "other terminal" can be "anywhere," if there is a composite return path for current to amplifier common, and if the closed-loop transfer characteristic is stable. The load is commonly represented as a single element connected to common, for simplicity. In Figure 4, the follower-connected output is equal to the voltage at the positive input, which is equal to the load voltage plus the battery voltage. Hence, the voltage across the resistor R_0 must be maintained equal to the battery voltage, and the current through the load must therefore be E_0/R_0, in the absence of error. In all circuits discussed here, the possible need for capacitance across feedback resistors, and (to a judicious degree) across the load, to preserve dynamic stability, is implicit. In addition to the usual current and voltage offset errors common mode error should also be considered and minimized, or compensated for.

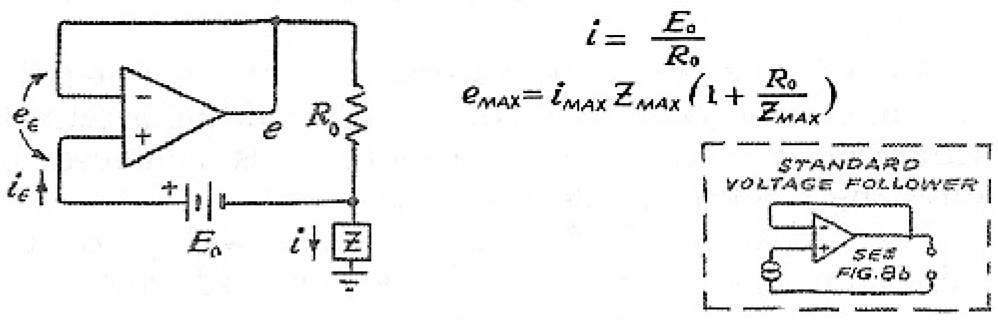


Fig. 4 Voltage to Current Transconductor (Floating source, grounded load, differential amplifier; source supplies no signal current)

The circuit of Figure 4 may be considered a degenerate case of the circuit of Figure 5a, which has fascinating properties. An equivalent circuit, which could employ two single-ended (conveniently chopper-stabilized) amplifiers, is shown in Figure 5b. This circuit, first described to us by Lincoln Laboratory, is known in our organization as the "Howland" Circuit. It can provide a current "to ground" proportional to the sum and difference of any number of grounded source voltages. Only one pair of such voltages is shown in Figure 5. Note that in the circuit of Figure 5a, the direct input supplies full current at short circuit, no current when $Z_L = R_0$, and negative current when Z_L is greater than R_0. The inverting input supplies i/K at short circuit, and increasing current as Z_L increases.

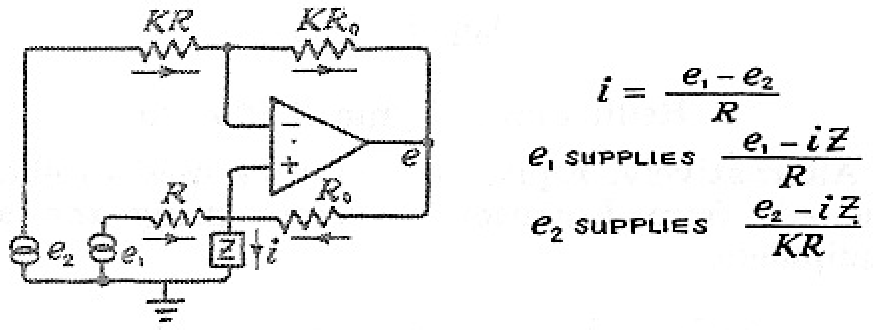


Fig. 5a Differential Voltage to Current Transducer (Grounded sources, grounded load, differential amplifier)

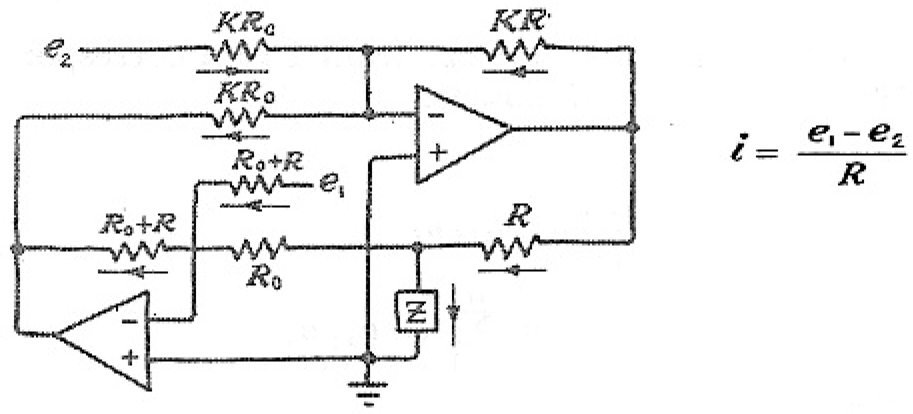


Fig. 5b Differential Voltage to Current Transducer (Grounded sources, grounded load, two single-ended amplifiers)

NEGATIVE IMPEDANCE

By definition, an ideal current source will have infinite impedance. One can easily satisfy oneself that all the current sources described above come quite close to meeting this requirement. Simply let the load impedance approach open circuit conditions, and observe that the load voltage strives toward saturation, even for quite small currents. But there are other interesting possibilities suggested by the circuit of Figure 5a (and its two-amplifier equivalent* Figure 5b).

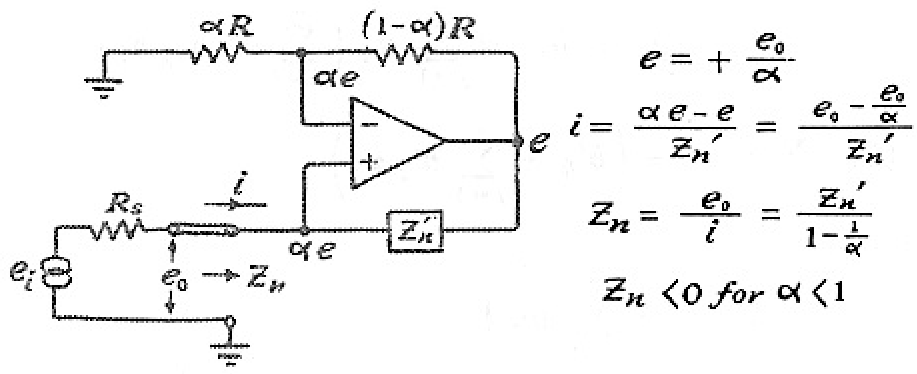


Fig. 6a Negative Impedance Circuit

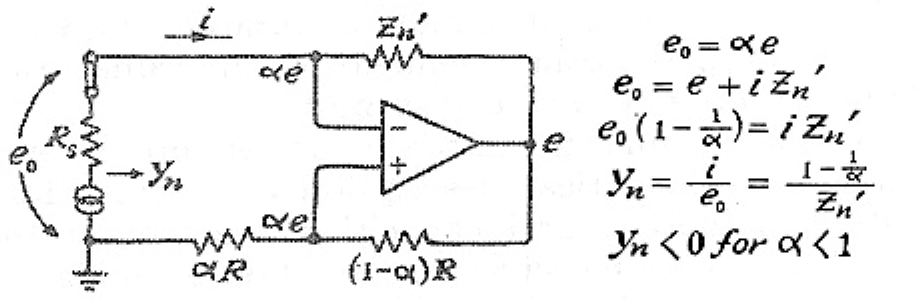


Fig. 6b Negative Admittance Circuit

These will be seen in Figure 6, in which are shown two varieties of negative impedance: in 6a the "short circuit stable* negative impedance" (or, more simply, *negative impedance*); and in 6b the "open circuit stable* negative impedance" (or, more simply, *negative admittance*). In both these circuits, it is possible to have negative resistance, capacitance, inductance, or com-

* Karplus, W. J., *Analog Simulation*, McGraw-Hill, 1958, pp. 257–259.

binations thereof. The circuit of Figure 6a will be unconditionally stable (assuming ideal amplifier and circuitry) if the magnitude of the source impedance in parallel with any shunt impedance to ground is always less than Z_n, and stable under some conditions if this magnitude is less but proper phase relations are maintained.

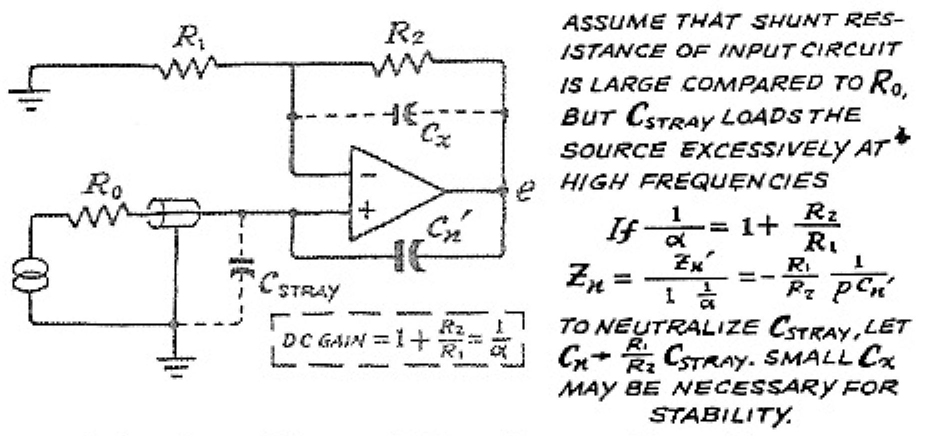


Fig. 6c Neutralizing Input Capacitance

One possibly useful and suggestive application is that of an active network having negative capacitance to compensate for lead capacitance in the amplifier input and associated circuitry. The elements of such a circuit are shown in Figure 6c. Although tailoring the dynamics of the amplifier may be necessary to insure stability, the neutralization of capacitance at the input can make possible high impedance measurements that would otherwise be all but unattainable. There are obviously many other applications for an accurate and flexible negative driving-point impedance or admittance.

It is interesting to note that the circuits of Figs. 6a and 6b resemble the Howland circuit with infinite impedance load, when the positive source impedances are equal to Z_n (i.e., at the point of instability).

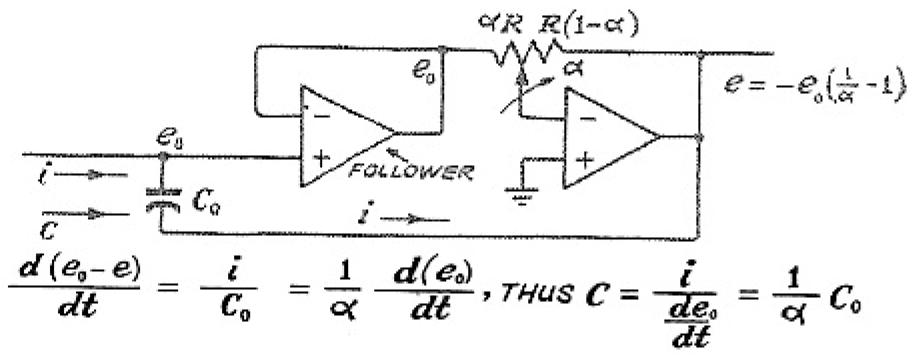


Fig. 7a Capacitance-Stretcher Circuit

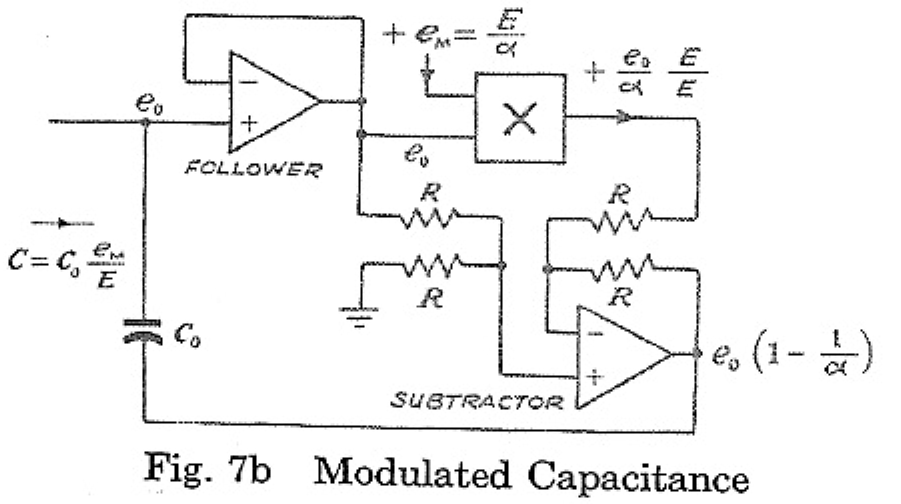


Fig. 7b Modulated Capacitance

MISCELLANEOUS TOPICS

1. This business of impedance transformation is not limited to positive to negative transformations alone. Consider the circuit of Figure 7a, in which we achieve a result opposed to that of Figure 6c, an *increase* in the value of a capacitance. Using this basic arrangement, we can linearly modulate the size of a capacitance with a voltage, as shown in Figure 7b, using an analog multiplier, or a servoed potentiometer. Because the amplifier has gain, circuit voltages must be such as not to drive the amplifier output into saturation.

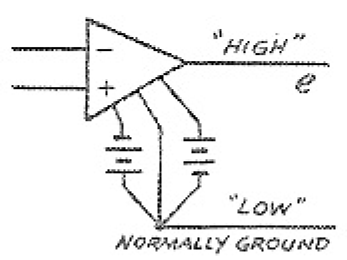


Fig. 8a
Operational Amplifier, Showing Differential Inputs and Outputs

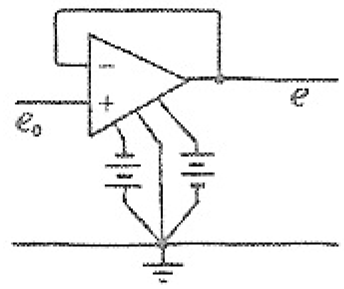


Fig. 8b
Conventional Follower

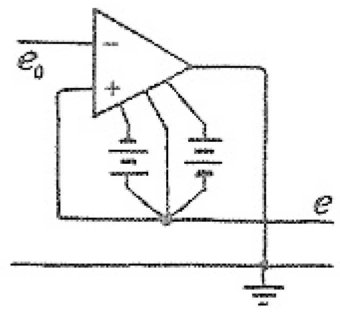


Fig. 9a
Inverted Follower (Could utilize chopper-stabilized amplifier)

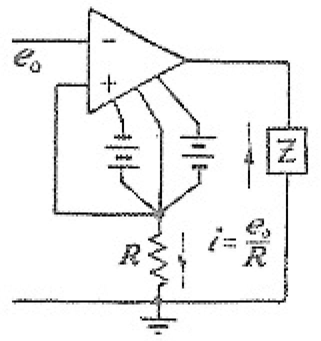


Fig. 9b
Inverting Amplifier as Voltage to Current Transconductor

2. A new dimension of flexibility can be gained (and lost) by allowing the power supply to float. (Separate power supplies are then needed for each amplifier.) The differential operational amplifier has available a negative and a positive input, and a positive and a negative output (See Figure 8). If either output terminal can be grounded, as well as either input terminal, new possibilities for measurement become available, particularly for chopper-stabilized amplifiers, in which the "plus" input and "minus" output are normally tied together irrevocably. For example, a stabilized follower having extremely high input impedance is practicable (Figure 9a). And in Figure 9b a current generator is shown, having grounded source and grounded load. In this instance, the grounded source draws no current: It is the inverted version of Figure 4. Verification of this relationship is left to the Reader, assuming the Reader has read to this point with sufficient admittance (not negative).

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